Effects of temporal frequency on phase-dependent sensitivity to heterochromatic flicker

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Introductio

A common technique for isolating the responses of chromatic channels is antiphase temporal modulation of two photometrically matched chromatic sources. The luminances of the two sources are equated by heterochromatic flicker photometry (HFP): the sources are flickered in antiphase, and the radiance of one source is adjusted to give minimal perceived flicker.\(^1\) It is often assumed that HFP nulls the response of the luminance channel, so that modulation thresholds for antiphase flicker reflect only activity of chromatic channels. However, several studies reported that once a HFP setting has been made, there is often residual flicker that can be reduced or eliminated by adjustments in the relative phase of the two sources.\(^2,5\) This finding suggests that the luminance channel can contribute to the detection of antiphase flicker even when the sources are photometrically equivalent.

While a number of models have been developed that describe the luminance and chromatic channels in terms of combinations of cone responses, the physiological substrate of the channels is uncertain. Smith et al.\(^7\) showed that temporal processing of chromatic stimuli can be studied without reference to a particular model of cone inputs to the channels. They measured colorimetric purity thresholds as a function of stimulus duration for wavelengths from 430 to 650 nm and fitted their data with a single template for temporal integration. This analysis uses the concept of a chromatic response with distinct temporal properties rather than assuming a particular model for the neural substrate of the chromatic channels.

Lindsey et al.\(^5\) used a similar approach to analyze modulation sensitivity for heterochromatic flicker of red and green primaries. They used sinusoidal modulation and measured modulation thresholds for a full range of relative phases of the primaries. They analyzed their data in terms of luminance and chromatic responses by decomposing their stimulus into luminance and chromatic components:

\[
L(t) = L + [M \cos \theta/2][\sin(ft/360) - (\theta/2)],
\]

\[
C(t) = C + [M \cos \theta/2][\cos(ft/360) - (\theta/2)],
\]

where \(L(t)\) and \(C(t)\) are the time-dependent and \(L\) and \(C\) the time-averaged luminance and chromaticity, respectively.\(^6\) This finding suggests that the luminance channel can contribute to the detection of antiphase flicker even when the sources are photometrically equivalent.

The approach of Lindsey et al. offers the following advantages over previous attempts to evaluate temporal properties of the chromatic response:

1. The analysis quantifies the luminance response to antiphase flicker. Once the phase shift is determined for a given frequency, the luminance response for any relative phase can be computed from the luminance response to inphase flicker. At 12 Hz the cosine template was maximum at 0° and minimum at 180°, indicating minimal luminance response to antiphase flicker. At 6 Hz, however, the luminance cosine template was smaller at a phase of 180° than at 180°, and the luminance response to antiphase flicker was larger than the chromatic response. Thus, even at frequencies at which the chromatic response is large, it may not be possible to isolate it with antiphase flicker.

2. The analysis allows evaluation of the combination rule for the chromatic and luminance responses. The stimuli have both chromatic and luminance components, so the data are highly sensitive to the combination rule used to generate predictions. Lindsey et al. evaluated their data by using different combination rules and found that vector summation gave the best fits. Modulation thresholds at 3 Hz were nearly independent of stimulus phase. This result
can be predicted by vector summation but not by linear summation or peak detection. Using vector summation, the modulation at threshold is given by

\[ M = \frac{1}{(|\cos^2(\theta - \phi)|/2)} + \sin^2((\theta - \phi)/2)M_{CT}^{-1/2}, \]

where \( \phi \) is the physiological phase shift, and \( M_{LT} \) and \( M_{CT} \) represent threshold modulation at phase settings that isolate luminance and chromatic components, respectively.

3) The analysis gives a more accurate estimate of the temporal sensitivity of the chromatic response. The analysis fits modulation-threshold data for a variety of phases (rather than just antiphase data), using the cosine templates for luminance and chromatic responses. Modulation thresholds for phases in which the luminance response is smallest give better isolation of the chromatic response than modulation thresholds for antiphase flicker. Although the analysis may still overestimate the chromatic response (for example, when the HFP match is imperfect), it will still be more accurate than an estimate based only on antiphase flicker data.

4) The analysis is not dependent on any specific models of luminance and chromatic channels in terms of cone inputs. The term "physiological phase shift" has traditionally been used to indicate a phase lag in the cone responses.\(^2\) The effect of such a phase shift on the luminance and chromatic responses cannot be computed without a specific model of chromatic and luminance channels in terms of cone inputs. Lindsey et al. introduced a new use of the term "phase shift": the shift of the luminance and chromatic cosine templates (along the phase axis) required to fit the data. By defining the phase shift in terms of the cosine templates, the analysis reveals properties that must hold for any successful model of luminance and chromatic channels.

Lindsey et al. collected data at four temporal frequencies, of which only one (6 Hz) showed a phase shift sufficient to cause a significant luminance response to antiphase flicker. Thus the generality of their result is not clear. In addition, with only four frequencies it is not possible to compare their values for chromatic and luminance sensitivity with previous estimates. We applied their technique to 13 frequencies: 1, 2.8, 4, 5, 6.2, 8, 10, 13.3, 16, 20, 26.7, and 40 Hz. A relative phase difference of 0° gives pure luminance modulation, a relative phase difference of 180° gives pure chromatic modulation, and intermediate relative phase differences give combined luminance and chromatic modulation. For any given block of trials, frequency was held constant and phase varied randomly from trial to trial. Data were gathered for 18 phases in 20° steps from \(-160°\) (the 625-nm LED leading the 644-nm LED) to \(160°\).

For the main data set, the field size was 2°. At the beginning of an experimental session, the 644-nm LED was set to 450 Td, and the observer matched the two sources by HFP, adjusting the radiance of the 625-nm LED to obtain a percept of minimum flicker. This setting was then used for the 625-nm LED throughout the session. The modulations of the two LED's were identical, and the mean luminance was 900 Td. The dominant wavelength of the field was determined by metamer matching to monochromatic stimuli. For observers WS and RS the field was metamer to 600 nm, and for observer RV it was metamer to 603 nm.

Three supplementary series of experiments were performed to evaluate the effect of change in field size, mean chromaticity, or photometric setting. In the first series, measurements were repeated with field sizes of either 8° or 0.5°. In the second series, the field size was fixed at 2°, and mean chromaticity was varied. We wished to change the mean chromaticity while maintaining the same average retinal illuminance of 900 Td. To shift the mean chromaticity to a longer wavelength, we set the mean luminance of the 564-nm LED to 300 Td (with 100% modulation) and matched this with the 625-nm LED (with 50% modulation) by HFP. Under these conditions, the luminance of the 564-nm LED varied by 600 Td (from 0 to 600 Td, about a mean of 300 Td), so presumably the HFP match resulted in the 625-nm LED's varying by 600 Td (from 300 to 900 Td, about a mean of 600 Td). When HFP was performed under these conditions, the resulting field appeared metamer to 610 nm. To shift the mean chromaticity to a shorter wavelength, we set the mean luminance of the 564-nm LED to 300 Td (with 100% modulation) and matched this with the 625-nm LED (with 50% modulation) by HFP. Under these conditions, the luminance of the 564-nm LED varied by 600 Td (from 300 to 900 Td, about a mean of 600 Td), so presumably the HFP match resulted in the 625-nm LED's varying by 600 Td (from 0 to 600 Td, about a mean of 300 Td). When HFP was performed under these conditions, the resulting field appeared metamer to 583-nm. In the third series of experiments, we tested the effect of intentional photometric mismatches. A frequency of 8 Hz was used, with three different settings for the radiance of the red LED, i.e., a normal HFP setting and settings 0.2 log unit above and below this setting. For each setting, three threshold measurements were made for 18 relative phases.

Three observers participated in these experiments. All three have normal color vision (Nagel anomaloscope and

METHODS

We used a computer-controlled two-channel Maxwellian-view system\(^{10}\) employing two LED's metamer to 564 and

625 nm, respectively. Observers viewed the stimulus in a darkened room. A chin rest was used to maintain a constant head position. This was deemed sufficient since dichromats were able to eliminate the percept of flicker for 8-Hz antiphase stimuli at their photometric matches. Modulation threshold was measured as a function of the relative phase difference of the sinusoidally modulated LED's for 13 frequencies: 1, 2, 2.8, 4, 5, 6.2, 8, 10, 13.3, 16, 20, 26.7, and 40 Hz. A relative phase difference of 0° gives pure luminance modulation, a relative phase difference of 180° gives pure chromatic modulation, and intermediate relative phase differences give combined luminance and chromatic modulation. For any given block of trials, frequency was held constant and phase varied randomly from trial to trial. Data were gathered for 18 phases in 20° steps from \(-160°\) (the 625-nm LED leading the 644-nm LED) to \(160°\).

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We fitted our data with the cosine templates for chromatic Farnsworth-Munsell 100-hue test. Observers RS and WS.

### Table 1. Modulation Thresholds for 13 Frequencies and 18 Relative Phases

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* Frequencies are given in hertz, and relative phases are given in degrees.

Farnsworth–Munsell 100-hue test). Observers RS and WS have normal visual acuity with their usual refractive correction, and observer RV has normal visual acuity without correction.

### DATA ANALYSIS

We fitted our data with the cosine templates for chromatic and luminance components described above. Each cosine template gives relative modulation threshold values as a function of stimulus phase. We defined the sensitivity of a cosine template as the reciprocal of threshold modulation at the template's phase of maximum sensitivity. In this analysis, differential phase shifts for the luminance and chromatic templates cannot be resolved, so the phase shifts for the two templates were assumed to be identical.

The following three parameters were available at each frequency to fit our data, using Eqs. (1)–(3): (1) the phase shift, (2) the sensitivity of the luminance cosine template, and (3) the sensitivity of the chromatic cosine template. The phase shift can be determined directly from the data. Once the phase shift is determined, the sensitivity of the more sensitive cosine template can be determined by fitting data near the phase of greatest sensitivity. The sensitivity of the less sensitive cosine template can then be determined by fitting the remaining data.

(1) At each frequency, the phase shift was determined from the data by deriving an axis of symmetry from threshold values interpolated in 1° steps. Each phase between 0° and 180° was evaluated as a potential axis of symmetry; the interpolated values were reflected about the potential...
axis of symmetry, and a sum of squared residuals was computed. The phase axis \((\theta)\) with the smallest sum of squared residuals was selected as the axis of best symmetry. Equations (1) and (2) show that this phase axis is equivalent to a phase axis \(180^\circ\) away \((\theta + 180^\circ)\); one of these two phases is the phase of least sensitivity (sensitivity of the least sensitive template) and the other is the phase of greatest sensitivity (sensitivity of the most sensitive template).

(2) Once a value was determined for the phase shift, the sensitivities of the chromatic and luminance cosine templates were determined for each temporal frequency. Near the phase of greatest sensitivity, threshold is determined by

**Fig. 1.** Predictions for modulation threshold as a function of the relative phase difference of the two LED's. Data shown are for frequencies from 1 to 40 Hz, with a 900-Td 2° field metameric to approximately 600 nm. Curves drawn through the data were derived by using the model of Lindsey et al.\(^8\). Left-hand panel shows data for subject WS, middle panel shows data for subject RS, and right-hand panel shows data for subject RV. For clarity, the data for each frequency were scaled vertically by the following additive scaling factors: 0.1 (2 Hz), 0.2 (2.8 Hz), 0.3 (4 Hz), 0.4 (5 Hz), 0.5 (6.2 Hz), 0.6 (8 Hz), 0.7 (10 Hz), 0.8 (13.3 Hz), 0.9 (16 Hz), 1.0 (20 Hz), 1.0 (26.7 Hz for subjects WS and RS), 1.1 (26.7 Hz for subject RV), 0.6 (40 Hz for subjects WS and RS), 0.9 (40 Hz for subject RV).
just one cosine template (the more sensitive one). We determined the sensitivity of the more sensitive cosine template by fitting it to data for phases within 40° of the phase of greatest sensitivity. For frequencies below 4 Hz the chromatic template was more sensitive, and for frequencies above 4 Hz the luminance template was more sensitive.

(3) Once the phase shifts and the sensitivity of one template were fixed, we fitted the entire data set by using a vector sum of the luminance and chromatic cosine templates, varying only the sensitivity of the remaining channel.

This analysis yielded values for the phase shift and for the temporal sensitivities of the chromatic and luminance templates.

RESULTS

The modulation-threshold data are given in Table 1 and are shown as filled triangles in Fig. 1. A similar pattern is seen for all three observers. For frequencies below 4 Hz, modulation thresholds are greater for relative phases near 0° (primarily luminance flicker) than for relative phases near 180° (primarily chromatic flicker). For frequencies above 5 Hz, modulation thresholds are greater for relative phases near 180° than for relative phases near 0°. At 4 Hz, modulation thresholds are nearly invariant with relative phase. For most frequencies above 5 Hz, the modulation thresholds are not symmetric about 0°, and, for frequencies below 6 and 10 Hz, modulation thresholds at 140° and 160° (stimulus condition red-leads-green) are greater than modulation thresholds at 180°. The solid lines in Fig. 1 are the predictions of the cosine templates by using Eqs. (1)–(3). In general, the predictions give good fits to the data. The phase shifts and sensitivities used for these predictions are given in Table 2.

Before the paper by Lindsey et al., phase shifts were evaluated by holding the modulation constant and adjusting the relative phase to determine the phase of least sensitivity. Instead, Lindsey et al. held the phase constant and varied the modulation. As described above, analysis of these data can also yield the phase of least sensitivity. In Fig. 2 we show the phase of least sensitivity as a function of temporal frequency. These functions are very similar for all three observers (left panel): The phase of least sensitivity is near 0° at 1 and 2 Hz, increases with frequency from 1 to 13 Hz, changes from stimulus condition red-leads-green to stimulus condition green-leads-red between 13 and 20 Hz, and then remains relatively constant from 20 to 40 Hz. In addition, for observer WS the phase of least sensitivity at 1 Hz is for stimulus condition green-leads-red, changing to stimulus condition red-leads-green between 1 and 2.8 Hz. The supplementary experiments showed that variations in field size (Fig. 2, middle panel) and chromaticity (Fig. 2, right panel) had little effect on the relation between temporal frequency and phase of least sensitivity.

The sensitivities of the luminance (filled symbols) and chromatic (open symbols) cosine templates are shown as functions of temporal frequency (Fig. 3). These modulation-sensitivity functions are of the same general shapes found in previous studies: the luminance functions are bandpass, and the chromatic functions are low pass. The solid and dotted lines show analytic modulation-sensitivity functions derived from the values for sensitivity of the luminance and chromatic cosine templates. To derive these analytic functions, we used a single low-pass filter to fit the sensitivities for the chromatic cosine template and a difference of two low-pass filters to fit the sensitivities for the luminance cosine template (Table 3).

With the chromaticities that we employed, it is difficult to perform HFP settings for a spatially homogeneous field at 900 Td, but the daily variations for our observers were less than 0.1 log unit. Intentional photometric mismatch had little effect on the phase shifts: the median thresholds are shown as a function of relative phase for each setting (Fig. 4). Threshold modulation at 160° is lowered by intentional photometric mismatch, but there is little change in the axis of symmetry. The axes of symmetry were calculated for each setting; they were 157° for the normal setting, 146° for the 0.2-log unit increase, and 153° for the 0.2-log unit decrease.

Table 2. Parameters Used to Generate the Predictions Shown in Fig. 1 for Our Three Observers

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<td>40.0</td>
<td>8</td>
<td>-</td>
<td>0.228</td>
<td>3</td>
<td>-</td>
<td>0.216</td>
<td>6</td>
<td>-</td>
<td>0.404</td>
</tr>
</tbody>
</table>

a For each frequency, values are given for the phase shift (Shift), sensitivity of the chromatic template (S1, in log units), and sensitivity for the luminance template (S2, in log units).
Fig. 2. Phase of least sensitivity as a function of temporal frequency. Left panel shows values for three observers with a 2°, 600-nm field. Middle panel shows values for observer WS with 600-nm fields of 0.5°, 2°, and 8°. Right panel shows values for observer WS with 2° fields metameric to 583, 600, and 610 nm.

Fig. 3. Values obtained for sensitivities of the luminance and chromatic cosine templates as a functions of frequency, with analytic modulation-sensitivity functions derived using the method of Swanson et al.\cite{10} Values for the parameters used to generate these functions are given in Table 2. Filled symbols are values for cosine-template sensitivities, and solid lines are the modulation-sensitivity functions for the luminance channel. Open symbols are scaling factors, and dashed lines are the modulation-sensitivity functions for the chromatic channel. Left panel is for subject WS, middle panel is for subject RS, and right panel is for subject RV.

| Table 3. Parameters for Modulation-Sensitivity Functions Shown in Fig. 3a |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Corner Frequency (Hz)       | Luminance                   | Chromatic                   | Latency (msec)              | Filter 1/Filter 2 Sensitivity Ratio\(^a\) |
| Observer                   | Cosine Template             | Filter 1                    | Filter 2                    |                             |
| WS                         | 11.3                        | 22.6                        | 16.7                        | 15                          | 1.54                        |
| RS                         | 18.8                        | 23.8                        | 19.4                        | 33                          | 1.06                        |
| RV                         | 16.2                        | 26.6                        | 24.9                        | 22                          | 1.39                        |

\(^a\) Discussed in Note 3.

\(^b\) The final column (Sensitivity Ratio) gives the ratio of the sensitivities of the two filters.
Phase-dependent sensitivity data at 900 Td reveal a systematic relation between temporal frequency and the phase of least sensitivity. This relation is similar for three observers and is robust for changes in field size and chromaticity.

Our range of 13 frequencies permitted evaluation of the frequency-dependent characteristics of the luminance and chromatic responses; the resulting functions are in agreement with those of previous studies.\(^8\) We show larger phase shifts than those reported by Lindsey \textit{et al.}\(^8\) because of our use of a 900 Td field. We gathered supplementary data on observer WS at 100 Td and found phase shifts similar to the values obtained by Lindsey \textit{et al.} Our method for deriving the phase of least sensitivity is model independent and rapid; when the data of Lindsey \textit{et al.} were analyzed, using our method, the resulting phase shifts were almost identical to the values that they obtained by varying all parameters of their model simultaneously.

Our data confirm and extend the findings of previous researchers. Cushman and Levinson\(^6\) evaluated phase shifts for frequencies of 20–55 Hz and obtained minimum sensitivity for the stimulus condition green-leads-red. Lindsey \textit{et al.}\(^8\) found minimum sensitivity for stimulus condition green-leads-red at 2 Hz and for stimulus condition red-leads-green at 6 Hz. The average values for our three observers at 900 Td are comparable with the average values for the three observers of Cushman and Levinson at 340 Td and those for the two observers of Lindsey \textit{et al.} at 100 Td (our Fig. 5). In our study, for frequencies above 16 and at 1 Hz, the phase of least sensitivity is for stimulus condition green-leads-red. From 2.8 to 16 Hz the phase of least sensitivity is for stimulus condition red-leads-green.

Two recent studies suggest that temporal sensitivity for antiphase flicker of photometrically matched sources (at retinal illuminances comparable with those in the present study) reflects contributions from both chromatic and luminance responses.\(^{16,17}\) Our data suggest the following explanation for this phenomenon: The phase of least sensitivity is other than 180° at most frequencies, giving a residual luminance response to antiphase flicker.

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**REFERENCES AND NOTES**

11. The reported phase axes were obtained by performing this analysis, using differences in the logarithms of the modulations. When the analysis was performed by using linear differences of the modulations, phase axes were within 2° of the reported values, with the following exceptions: 5° higher for subject WS at 1 Hz, 5° higher for subject RV at 10 Hz, 3° lower for subject RS at 16 Hz, 8° lower for subject WS at 5 Hz, 12° higher for subject RS at 4 Hz, and 15° lower for subject RV at 4 Hz.
12. The phase of least sensitivity for a given cosine template is directly related to the phase shift. With a phase shift of $\theta$, the chromatic response is smallest for a relative phase of $\theta$, and the luminance response is smallest for a relative phase of $\theta - 180^\circ$. The more sensitive cosine template determines threshold for all relative phases, except those near its phase of minimum response, so the phase of least sensitivity is determined by the more sensitive channel.
13. We followed the convention of Cushman and Levinson, who used the term “green-leads-red,” to refer to the shift in response onset from a relative phase of $180^\circ$. For our stimuli, the term “green-leads-red” refers to relative phases from $-160^\circ$ to $0^\circ$, and “red-leads-green” refers to relative phases from $0^\circ$ to $180^\circ$.
14. Data were not gathered at 40 Hz for the 0.5° field nor for the 2° fields metameric to 610 and 583 nm, since flicker could not be detected with the maximum available modulation under these conditions.
15. For observer WS, the luminance modulation-sensitivity function shown was derived previously from data gathered on the same apparatus; see Ref. 10. The remaining modulation-sensitivity functions were derived in the same manner from the values obtained for sensitivities of the luminance and chromatic cosine templates. First, a digital impulse-response function was derived by using the method of Stork and Falk. This digital function was then fitted with an analytic function. The analytic modulation-sensitivity function is the Fourier transform of this analytic impulse-response function. For chromatic sensitivities, the analytical functions were five-stage linear filters; for luminance sensitivities the analytical functions were differences of two five-stage linear filters (response of the second filter was delayed by a latency). The parameters for these functions are shown in Table 3.